$\boxed{\text{Solutions}}$ (-1) for signs.

Math 2E Quiz 2 Afternoon - April 7th

Please put ID on back for redistribution!

Show all work, and try to simplify your answers. *There is a question on the back side.

1. [10pts] You will be finding the volume of the solid enclosed by the following surfaces:

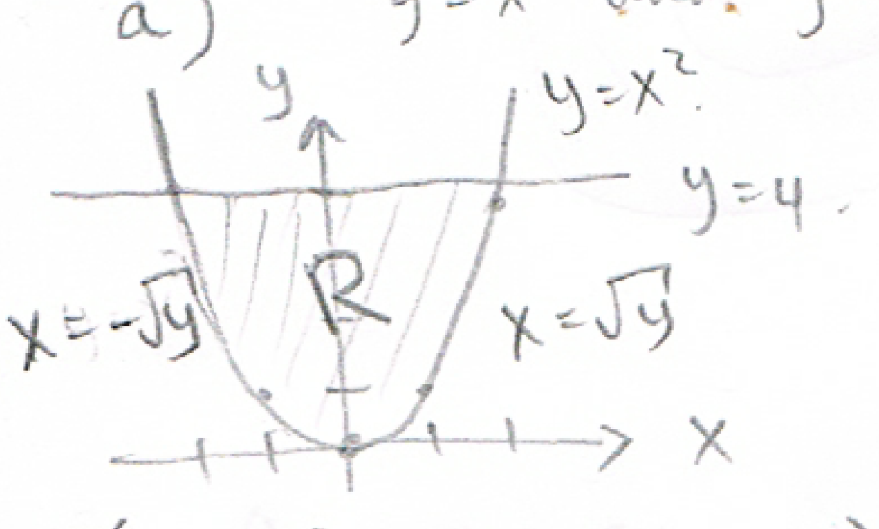
The "cylinders" (sheets) $z = x^2, y = x^2$ and the planes $z = 0, y = 4$.

(Recall that "cylinders" are really just sheets - it's a bit of a misnomer.)

(a) Write the integral that gives the volume as both $\iint_R f(x,y)\, dy\, dx$ and $\iint_R f(x,y)\, dx\, dy$.

(b) Compute the volume using one of the two integrals from (a).

a) $y = x^2$ and $y = 4$ yield the region R; z ranges from 0 to x^2.



$(y = x^2 \Rightarrow x = \pm\sqrt{y})$

$$\Rightarrow \int_{x=-2}^{x=2}\int_{y=x^2}^{y=4} (x^2 - 0)\, dy\, dx \quad +3$$

$$\int_{y=0}^{y=4}\int_{x=-\sqrt{y}}^{x=\sqrt{y}} (x^2 - 0)\, dx\, dy \quad +3$$

b) $\int_{-2}^{2}\int_{x^2}^{4} x^2\, dy\, dx$

$= \int_{-2}^{2} x^2 y \Big|_{x^2}^{4} dx$ +2

$= \int_{-2}^{2} (4x^2 - x^4)\, dx$; The fns are even!

$= 2\left(\frac{4x^3}{3} - \frac{x^5}{5}\right)\Big|_0^2$; $4 = 2^2$, so

$= 2\left[\frac{2^5}{3} - \frac{2^5}{5}\right]$

$= 2^6\left(\frac{1}{3} - \frac{1}{5}\right)$; $\frac{5}{15} - \frac{3}{15} = \frac{2}{15}$

$= \boxed{\frac{2^7}{15}}$ +2

$(2^7 = 128)$

OR

$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} x^2\, dx\, dy$; x^2 is an even function

$= \int_0^4 2 \cdot \frac{x^3}{3}\Big|_0^{\sqrt{y}} dy$

$= \int_0^4 \frac{2}{3} y^{3/2}\, dy$ +2

$= \frac{2}{3} \cdot \frac{2y^{5/2}}{5}\Big|_0^4$

$= \frac{4 \cdot 4^{5/2}}{15}$

$= \frac{4^{7/2}}{15} = \frac{(4^{1/2})^7}{15}$ +2

$= \boxed{\frac{2^7}{15}}$

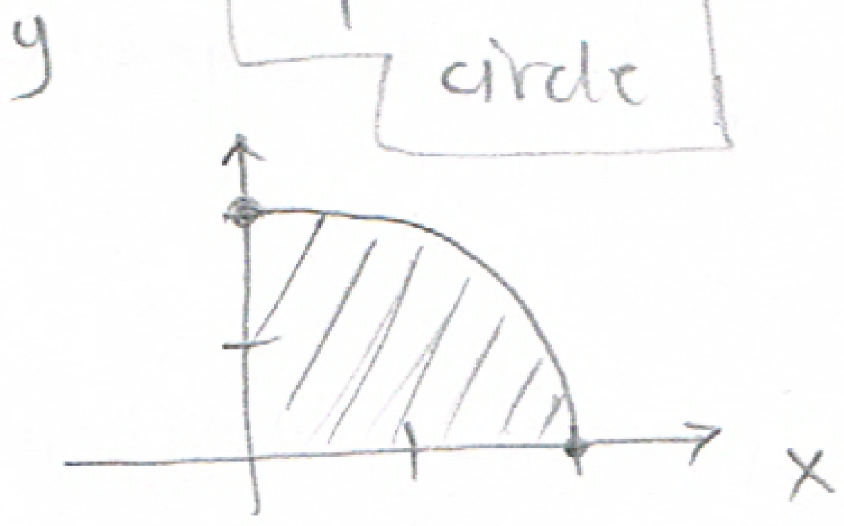
2. [10pts] Evaluate the integral by converting to polar coordinates:

$$\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy\, dx.$$

It should help to first draw the domain of integration.

$\frac{1}{4}$ of a circle

↳ • From domains of integrals, $0 \le y \le \sqrt{4-x^2}$, $0 \le x \le 2$ $\Rightarrow$



• $e^{-x^2-y^2}$ in polar becomes e^{-r^2} since $x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2 = r^2$.

• For domain, $0 \le r \le 2$, and θ goes from 0 to $\pi/2$

$\Rightarrow$ Integral Equals $\displaystyle\int_0^{\pi/2}\int_0^2 e^{-r^2}\, r\,dr\,d\theta$. +5

let $\boxed{u = r^2,\ du = 2r\,dr}$ $\longrightarrow$ $\displaystyle = \int_0^{\pi/2}\int_{u=0}^{u=4} e^{-u}\cdot\frac{du}{2}\,d\theta$

$$= \frac{1}{2}\int_0^{\pi/2} -e^{-u}\Big|_0^4\, d\theta$$

+4

$$= \frac{1}{2}\int_0^{\pi/2} (-e^{-4}+1)\, d\theta$$ // These are constants wrt. θ,

$$= \frac{1-e^{-4}}{2}\int_0^{\pi/2} d\theta \;;\quad \int_0^{\pi/2} d\theta = \frac{\pi}{2},\ \text{so}$$

$$= \boxed{\frac{\pi(1-e^{-4})}{4}}$$ +1